

# Gravity Probe B Experiment in 7D Space-and-Time Continuum

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## Abstract

This study deals with application of field equations in seven-dimensional space-and-time continuum to calculate geodetic and orbital gyroscope precession. It was demonstrated that unlike the classic theory the assumptions made completely correspond to the Gravity Probe B findings.

## 1 Introduction

Gravity Probe B is a space mission of the USA aimed at measuring extremely low effects of geodetic precession of the gyroscopes on circumterrestrial orbit which are predicted by Einstein's relativity theory. The satellite was launched on April 20, 2004 while data acquisition process commenced in August 2004. The satellite was for total 17 months on its orbit and completed its mission on October 3, 2005. Reduction was completed in May 2011. The experimental findings were published in the final report [1]. The geodetic gyroscope precession effect coincided with the estimated value with accuracy of 0.1%. The effect caused by the interaction between spin and orbital moment differs from the estimated value for more than 19%. This study suggests excluding the effect caused by spinning of bodies from the design equations.

## 2 Main part

As shown in equations [2] the dynamics of both translation and spin motion of bodies in gravity fields may be explained using seven-dimensional space-and-time continuum which in addition to time and three spatial coordinates

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comprises three coordinates that orient a body in space and may be described as Euler angles  $x^4 = \varphi$ ,  $x^5 = \psi$ ,  $x^6 = \theta$ . For flat space the metrics of a spherical body in an empty seven-dimensional space-and-time continuum is given by

$$\begin{aligned} g_{00} &= 1, & g_{\alpha\alpha} &= -1, & g_{44} &= -\frac{J}{m}, \\ g_{45} &= g_{54} = -\frac{J \cos(\theta)}{m}, \\ g_{55} &= -\frac{J}{m}, & g_{66} &= -\frac{J}{m}, \end{aligned} \quad (1)$$

where  $J$  is the sample body inertia relative to spin axes, precession and nutation,  $m$  is the body mass,  $\alpha = 1, 2, 3$ .

Equations [2] show that metrics (1) allows for obtaining three classical equations of sample spherical body motion in empty space

$$a_x = 0, \quad a_y = 0, \quad a_z = 0, \quad (2)$$

and three gyroscope equations which cannot be derived by the general four-dimensional relativity theory

$$\begin{aligned} \varepsilon_\varphi &= \frac{\omega_\psi \omega_\theta}{\sin \theta} - \frac{\cos \theta}{\sin \theta} \omega^4 \omega_\theta, \\ \varepsilon_\psi &= \frac{\omega^4 \omega_\theta}{\sin \theta} - \frac{\cos \theta}{\sin \theta} \omega_\psi \omega_\theta, \\ \varepsilon_\theta &= -\omega^4 \omega_\psi \sin \theta. \end{aligned} \quad (3)$$

Gravity equations in seven-dimensional space-and-time continuum may be written as

$$R_{mn} = \varkappa(T_{mn} - \frac{1}{2}g_{mn}T) + \Lambda_{mn}, \quad (4)$$

where  $\Lambda_{mn}$  is supplementary tensor [2]. Variable  $\Lambda_{mn}$  must be introduced because not all components of  $R_{mn}$  with metrics (1) are reduced to zero in the absence of matter  $T_{mn} = 0$ .

Let us project the body spin vector  $\omega^4$  on the coordinate space

$$\vec{\omega}^4 = \omega_\alpha^4 dx^\alpha$$

and consider translation of vector  $\vec{\omega}^4$  along the geodetic coordinate

$$\frac{d\omega_{\alpha}^4}{dt} = \Gamma_{\alpha\beta}^{\mu} u^{\beta} \omega_{\mu}^4, \quad (5)$$

where  $\alpha, \beta, \mu = 1, 2, 3$ .

As it is shown in [3] expand field equation (4) into series by velocity exponents. The, using translation equation (5) we derive the gyroscope angular velocity variation equation:

$$\frac{d\vec{\omega}^4}{dt} = \vec{\Omega} \times \vec{\omega}^4, \quad (6)$$

where

$$\vec{\Omega} = \frac{c}{2}(\vec{\nabla} \times \vec{\xi}) - \frac{3}{2}(\vec{u} \times \vec{\nabla} \phi) \quad (7)$$

is angular precession velocity,  $\phi$  and  $\xi$  are gravity potentials. Herewith due to orthogonality of its components the angular precession velocity may be expressed as the sum of two orthogonal vectors

$$\vec{\Omega}' = \vec{\Omega}' + \vec{\Omega}'',$$

where

$$\vec{\Omega}' = -\frac{3}{2}(\vec{u} \times \vec{\nabla} \phi)$$

is angular precession velocity,

$$\vec{\Omega}'' = \frac{c}{2}(\vec{\nabla} \times \vec{\xi})$$

is angular velocity of spin-spin interaction of the gyroscope and the Earth. Thus, the angular acceleration of the gyroscope may be expressed as

$$\varepsilon' = |\vec{\Omega}' \times \vec{\omega}^4|, \quad (8)$$

$$\varepsilon'' = |\vec{\Omega}'' \times \vec{\omega}^4|. \quad (9)$$

Alternatively, using gyroscope equation (3) with the assumption that the gyroscope angular velocity is  $\omega^4 = \text{const}$  it is possible to derive the equations for angular precession acceleration

$$\varepsilon_\psi = \omega^4 \omega_\theta \sin \theta, \quad (10)$$

and angular nutation acceleration

$$\varepsilon_\theta = -\omega^4 \omega_\psi \sin \theta. \quad (11)$$

Using the properties of spin, angular precession and angular nutation velocity vectors [2], and comparing equations (8), (9), (10) and (11) we discover that

$$\omega_\psi = -\frac{\Omega'}{\sin \theta}, \quad (12)$$

$$\omega_\theta = \Omega''. \quad (13)$$

Henceforth we deem values  $\omega_\psi$  and  $\omega_\theta$  obtained in Gravity Probe B experiment to be angular velocities of the gyroscope axis displacement.

In the performance of Gravity Probe B experiment the angular precession velocity was expected to be orthogonal to the gyroscope spin velocity ( $\theta = \pi/2$ ). But should it be wrong than due to recession it is impossible to uniquely determine by a small motion arc of spin axis  $AB$  if the arc occurs exclusively due to nonorthogonality of precession or due to a complex motion of orthogonal precession  $AB'$  and orthogonal nutation  $B'B$  (see Fig. 1). That is, the angular precession velocity at may be expended into two orthogonal components

$$\vec{\omega}_\psi = \vec{\omega}'_\psi + \vec{\omega}'_\theta$$

of virtual precession velocity  $\vec{\omega}'_\psi$  and virtual nutation velocity  $\vec{\omega}'_\theta$ .

If we determine the function of virtual precession  $\omega'_\psi$  and virtual nutation  $\omega'_\theta$  to actual precession  $\omega_\psi$  and nutation angle  $\theta$  (see Fig. 1) we can derive the equation

$$\omega'_\psi = \frac{\omega_\psi (\sin \theta)^2 \sin \psi}{\sqrt{1 - (2(\sin(\psi/2))^2 (\sin \theta)^2 - 1)^2}}, \quad (14)$$

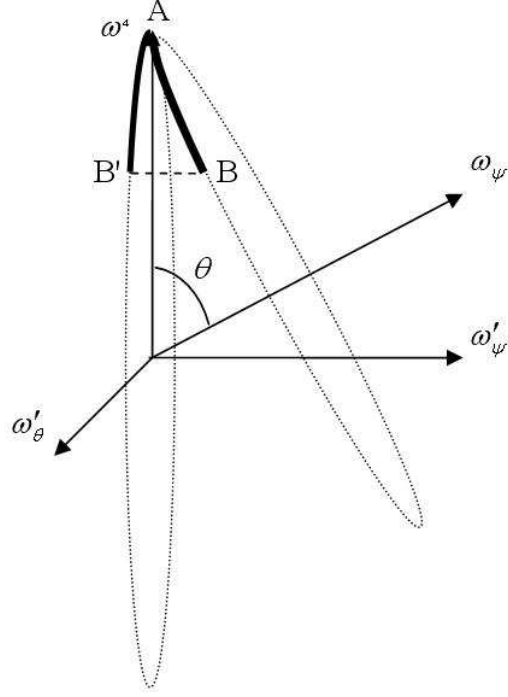


Figure 1: Angular velocity vectors of spin  $\omega^4$ , precession  $\omega_\psi$  and virtual angular velocity vectors of precession  $\omega'_\psi$  and nutation  $\omega'_\theta$

$$\omega'_\theta = \frac{\omega_\psi \sin \theta \sin \psi (1 - 2(\sin(\theta/2))^2)}{1 - \sin \theta (\sin \psi)^2}. \quad (15)$$

Substituting (12) into resulting relations (14) and (15), we have

$$\omega'_\psi = \frac{\Omega' \sin \theta \sin \psi}{\sqrt{1 - (2(\sin(\psi/2))^2(\sin \theta)^2 - 1)^2}}, \quad (16)$$

$$\omega'_\theta = \frac{\Omega' \sin \psi (1 - 2(\sin(\theta/2))^2)}{1 - \sin \theta (\sin \psi)^2}. \quad (17)$$

Therefore, we maintain that

$$\Omega'_\psi = \omega'_\psi, \quad (18)$$

$$\Omega''_\theta = \omega'_\theta + \omega_\theta. \quad (19)$$

were measurable values in Gravity Probe B experiment.

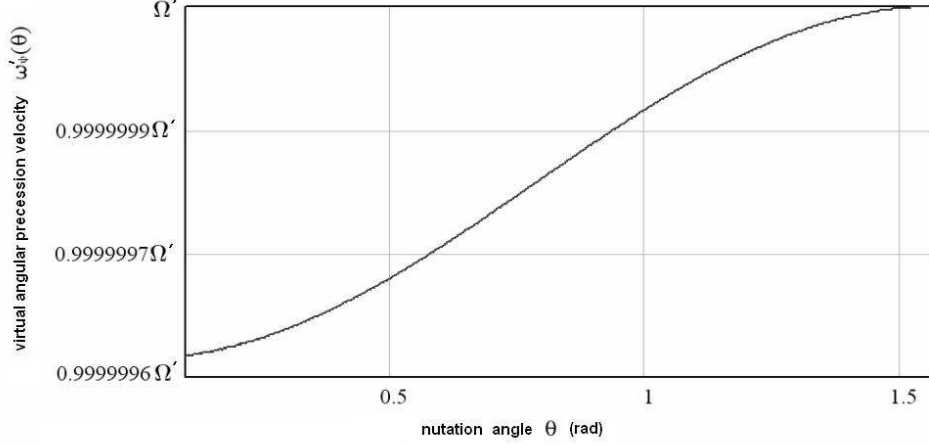


Figure 2: Function of virtual angular velocity of precession  $\omega'_\psi$  to nutation angle  $\theta$

As it appears from [1] the angular velocity of spin-spin interaction differs from the actual value for about 19%. The functions (Fig. 2, Fig. 3) show that the discrepancy between the theoretical and the experimental data may be avoided if we assume that the angular nutation velocity is contributed by misinterpretation of the angular precession velocity, that is the difference of nutation angle  $\theta$  from  $\pi/2$ . In particular, at  $\theta \approx 1.3965$  radian the estimated virtual angular velocity of nutation is  $\omega'_\theta = 3.027 \cdot 10^{-4}\Omega'$  which is equal to  $\omega'_\theta \approx 2$  angular milliseconds per year. Thus, if the measurable angular velocity of nutation  $\Omega'_\theta$  is equal to the sum of a theoretical value of angular nutation velocity  $\omega_\theta$  (refer to (13)) and a virtual value of angular precession velocity  $\omega'_\theta$  than the estimated value shall be exactly the same as the measurable value. Deviation of angular precession velocity  $\Omega'_\psi$  from the estimated value at nutation angle  $\theta \approx 1.3965$  radian will not exceed  $10^{-6}\%$ .

### 3 Conclusion

Finally it should be noted that considering the Gravity Probe B experiment in terms of a seven-dimensional gravitation model it is possible to obtain unorthogonal direction of the angular precession velocity relatively to the gyroscope angular spin velocity. This fact allows for interpreting the gyroscope spin axis motion as virtual precessional and virtual nutation motions. In view of the virtual angular nutation velocity contribution we discover that

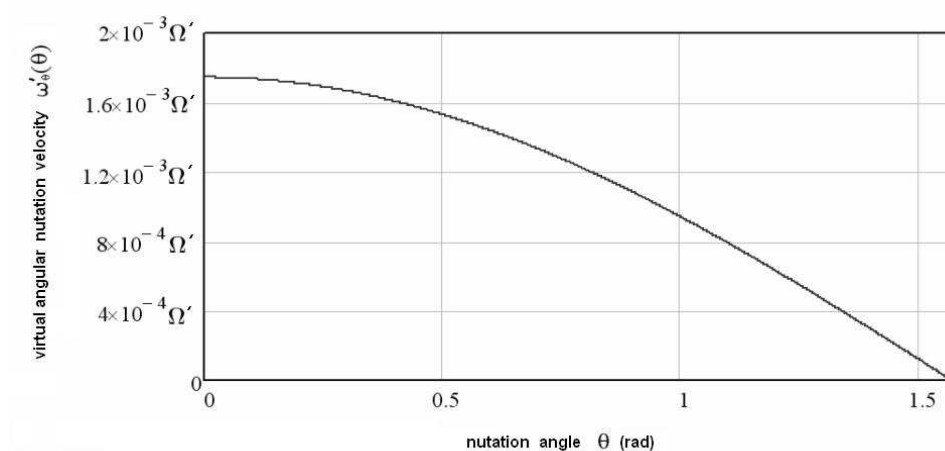


Figure 3: Function of virtual angular velocity of nutation  $\omega'_\theta$  to nutation angle  $\theta$

the experimental and estimated values of angular nutation velocity coincide at the particular angle  $\theta$ .

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## References

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